Chapter 13: Problem 1

The equation for the security market line is:

\[ \bar{R}_i = R_f + (\bar{R}_m - R_f) \beta_i \]

Thus, from the data in the problem we have:

\[ 6 = R_f + (\bar{R}_m - R_f) \times 0.5 \] for asset 1
\[ 12 = R_f + (\bar{R}_m - R_f) \times 1.5 \] for asset 2

Solving the above two equations simultaneously, we find \( R_f = 3\% \) and \( \bar{R}_m = 9\% \). Using those values, an asset with a beta of 2 would have an expected return of:

\[ 3 + (9 - 3) \times 2 = 15\% \]

Chapter 13: Problem 2

Given the security market line in this problem, for the two stocks to be fairly priced their expected returns must be:

\[ \bar{R}_x = 0.04 + 0.08 \times 0.5 = 0.08 \text{ (8\%)} \]
\[ \bar{R}_y = 0.04 + 0.08 \times 2 = 0.20 \text{ (20\%)} \]

If the expected return on either stock is higher than its return given above, the stock is a good buy.

Chapter 13: Problem 3

Given the security market line in this problem, the two funds' expected returns would be:

\[ \bar{R}_a = 0.06 + 0.19 \times 0.8 = 0.212 \text{ (21.2\%)} \]
\[ \bar{R}_b = 0.06 + 0.19 \times 1.2 = 0.288 \text{ (28.8\%)} \]
Comparing the above returns to the funds’ actual returns, we see that both funds performed poorly, since their actual returns were below those expected given their beta risk.

Chapter 13: Problem 4

Given the security market line in this problem, the riskless rate equals 0.04 (4%), the intercept of the line, and the excess return of the market above the riskless rate (also called the “market risk premium”) equals 0.10 (10%), the slope of the line. (The return on the market portfolio must therefore be 0.04 + 0.10 = 0.14, or 14%)

Chapter 13: Problem 5

The price form of the CAPM’s security market line equation is:

\[ P_i = \frac{1}{r_F} \left[ \overline{Y_i} - (\overline{Y_m} - r_F \times P_m) \times \frac{\text{cov}(Y_i, Y_m)}{\text{var}(Y_m)} \right] \]

where \( r_F = (1 + r_F) \) and \( \overline{R_m} = \frac{\overline{Y_m} - P_m}{P_m} \).

From Problem 4, we have \( r_F = 0.04 \) and \( \overline{R_m} = 0.14 \). Therefore \( 0.14 = \frac{\overline{Y_m} - P_m}{P_m} \) which gives \( 1.14P_m = \overline{Y_m} \).

Substituting these values into the above security market line equation, we have:

\[ P_i = \frac{1}{1.04} \left[ \overline{Y_i} - (1.14 \times P_m - 1.04 \times P_m) \times \frac{\text{cov}(Y_i, Y_m)}{\text{var}(Y_m)} \right] \]

\[ = \frac{1}{1.04} \left[ \overline{Y_i} - 0.10 \times P_m \times \frac{\text{cov}(Y_i, Y_m)}{\text{var}(Y_m)} \right] \]
Chapter 13: Problem 6

To be rigorous, one should use the four Kuhn-Tucker conditions shown in Appendix E of Chapter 6. To find the optimum portfolio when short sales are not allowed, we have, for each asset \( i \), the following Kuhn-Tucker conditions:

\[
\frac{d \theta}{dX_i} + U_i = 0 \quad (1)
\]

\[
X_i U_i = 0 \quad (2)
\]

\[
X_i \geq 0 \quad (3)
\]

\[
U_i \geq 0 \quad (4)
\]

We have already seen that, given the assumptions of the standard CAPM, setting \( \frac{d \theta}{dX_i} = 0 \) gives the equilibrium first order condition for asset \( i \), which is the standard CAPM’s security market line:

\[
\bar{R}_i = R_f + \left( \bar{R}_m - R_f \right) \beta_i
\]

or equivalently

\[
\bar{R}_i - R_f - \left( \bar{R}_m - R_f \right) \beta_i = 0
\]

When short sales are not allowed, Kuhn-Tucker condition (1) implies that:

\[
\bar{R}_i - R_f - \left( \bar{R}_m - R_f \right) \beta_i + U_i = 0
\]

But, since all assets are held long in the market portfolio, \( X_i > 0 \) for each asset and therefore, given Kuhn-Tucker condition (2), \( U_i = 0 \) for each asset. Thus, the standard CAPM holds even if short sales are not allowed.
Chapter 13: Problem 7

Using the two assets in Problem 1, a portfolio with a beta of 1.2 can be constructed as follows:

\[ 0.5X_1 + (1.5)(1 - X_1) = 1.2 \]

\[ X_1 = 0.3; \ X_2 = 0.7 \]

The return on this combination would be:

\[ 0.3(6\%) + 0.7(12\%) = 10.2\% \]

Asset 3 has a higher expected return than the portfolio of assets 1 and 2, even though asset 1 and the portfolio have the same beta. Thus, buying asset 3 and financing it by shorting the portfolio would produce a positive (arbitrage) return of

\[ 15\% - 10.2\% = 4.8\% \]

with zero net investment and zero beta risk.

Chapter 13: Problem 8

The security market line is:

\[ \bar{R_i} = R_f + (\bar{R_m} - R_f)\beta_i \]

Substituting the given values for assets 1 and 2 gives two equations with two unknowns:

\[ 9.4 = R_f + (\bar{R_m} - R_f) \times 0.8 \]

\[ 13.4 = R_f + (\bar{R_m} - R_f) \times 1.3 \]

Solving simultaneously gives:

\[ R_f = 3\%; \ \bar{R_m} = 11\% \]

Chapter 13: Problem 9

Substituting the given betas in the given equation yields:

\[ \bar{R}_1 = 0.178 \ (17.8\%); \ \bar{R}_2 = 0.151 \ (15.1\%) \]